Equitable distribution of resources in Transportation Networks

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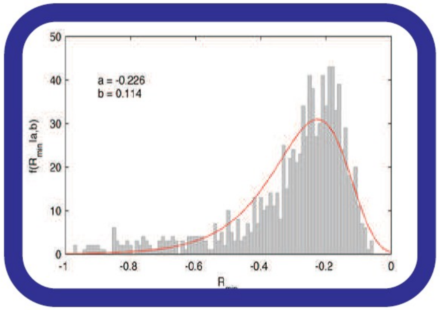
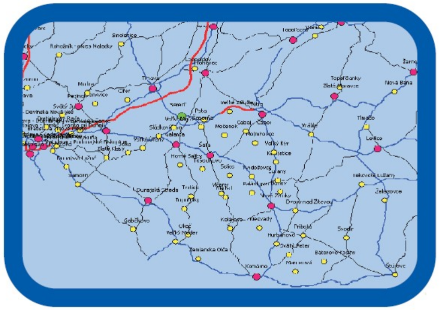
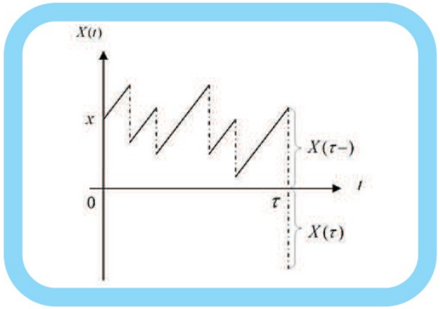
Education: Math courses for engineers (Algebra, Mathematical Analysis, Graph Theory, Game theory), **Optimisation Methods**, Data structures, Computer simulations, Geographical Information Systems, Cryptography,

Research: Optimisations and simulations of complex technological systems (private and public service systems, transportation networks, production systems, crisis management)







University of Žilina, University Science park
ERA chair in Intelligent Transport Systems

Chair holder: Dr. Karl Ernst Ambrosh

FP 7 Project: ERAdiate **Web:** *http://www.erachair.uniza.sk/*

Research fields:

1. Cooperative ITS *(eCall, localisation, standardisation)*

2. Decarbonisation of Mobility (eMobility, GHG indicators, modal shift, simulation, mathematical modelling)

3. Urban Mobility / Smart City *(urban mobility indicators, sensor networks, incident detection, urban logistics)*

4. Intermodal ITS (modal shift, big data, MaaS, SUMP, modelling and simulation)

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Equitable distribution of resources in Transportation Networks

1. Theoretical background

2. Application I: Resilience of Natural Gas Networks During Conflicts, Crises and Disruptions

3. Application II: Design of Public Service Systems

4. Application III: Coordination of EVs Charging in the Distribution Networks

5. Current and future work

1. Theoretical background

We consider a set of **users** J. User j assigns to a **decision vector** x a **utility** $U_j(x)$.

User utilities form the performance vector:

$$U(x)=(U_1(x),U_2(x),\dots,U_n(x))$$

Let $u_j=U_j(x)$ for $j=1,\dots,n$ and in vector notation $\boldsymbol{u}=\boldsymbol{f}(\boldsymbol{x})$ for Let the feasible outcome vectors be $\boldsymbol{U}$.

Terminology:

$\boldsymbol{u}^1 \preccurlyeq \boldsymbol{u}^2$ - $\boldsymbol{u}^1$ is weakly preferred over $\boldsymbol{u}^2$

$\boldsymbol{u}^1 \prec \boldsymbol{u}^2$ - $\boldsymbol{u}^1$ is strictly preferred over $\boldsymbol{u}^2$

$\boldsymbol{u}^1 \simeq \boldsymbol{u}^2$ - $\boldsymbol{u}^1$ and $\boldsymbol{u}^2$ are equally preferred

[H.Luss, Equitable Resource Allocation: Models, Algorithms and Applications, John Wiley & Sons, 2012]

1. Theoretical background

Equitable solutions (fairness schemes) should (ideally) satisfy the following properties (axioms):

1. *Completeness*: either $\boldsymbol{u}^1 \preccurlyeq \boldsymbol{u}^2$ or $\boldsymbol{u}^2 \preccurlyeq \boldsymbol{u}^1$ for any $\boldsymbol{u}^1, \boldsymbol{u}^2 \in \boldsymbol{U}$
2. *Transitivity*: If $\boldsymbol{u}^1 \preccurlyeq \boldsymbol{u}^2$ and $\boldsymbol{u}^2 \preccurlyeq \boldsymbol{u}^3$, then $\boldsymbol{u}^1 \preccurlyeq \boldsymbol{u}^3$ for any $\boldsymbol{u}^1, \boldsymbol{u}^2, \boldsymbol{u}^3 \in \boldsymbol{U}$
3. *Strictly monotonic*: $\boldsymbol{u} - \epsilon \boldsymbol{e}_j \prec \boldsymbol{u}$ for any $(\boldsymbol{u} - \epsilon \boldsymbol{e}_j), \boldsymbol{u} \in \boldsymbol{U}$ and $j = 1, \ldots, n$, where $\boldsymbol{e}_j$ is the *j*th unit vector and ϵ is an arbitrary small positive constant.
4. *Scale invariance*: If $\boldsymbol{u}^1 \preccurlyeq \boldsymbol{u}^2$, then $c\,\boldsymbol{u}^1 \preccurlyeq c\,\boldsymbol{u}^2$ for any $\boldsymbol{u}^1, \boldsymbol{u}^2 \in \boldsymbol{U}$ and $c > 0$.
5. *Anonymity (Impartiality)*: $\boldsymbol{u}^1 \simeq \boldsymbol{u}^2$ if $\boldsymbol{u}^1$ is the permutation of the elements of $\boldsymbol{u}^2$ for any $\boldsymbol{u}^1, \boldsymbol{u}^2 \in \boldsymbol{U}$

[H.Luss, Equitable Resource Allocation: Models, Algorithms and Applications, John Wiley & Sons, 2012]

1. Theoretical background

Equitable solutions (fairness schemes) should (ideally) satisfy the following properties (axioms):

6. *Principle of Transferability:* If $u_{j_1} > u_{j_2}$, then $\boldsymbol{u} - \epsilon\, \boldsymbol{e}_{\boldsymbol{j}_1} + \epsilon\, \boldsymbol{e}_{\boldsymbol{j}_2} \prec u$ for any $0 < \epsilon < u_{j_1} - u_{j_2}$ and $\left(\boldsymbol{u} - \epsilon\, \boldsymbol{e}_{\boldsymbol{j}_1} + \epsilon\, \boldsymbol{e}_{\boldsymbol{j}_2}\right), \boldsymbol{u} \in U$.

[H.Luss, Equitable Resource Allocation: Models, Algorithms and Applications, John Wiley & Sons, 2012]

1. Theoretical background

Fairness can be captured by combining a family of user utility functions, called α -fair:

$$U_j(x_j,\alpha)=\begin{cases}\dfrac{x_j^{1-\alpha}}{1-\alpha}\ for\ \alpha\geq 0,\alpha\neq 0\\ \log(x_j)\ \alpha=1\end{cases}$$

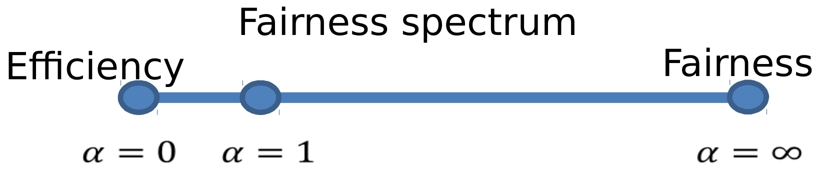
with optimisation:

maximise	$\sum_{j=1}^{n} U_j(x,\alpha)$
Subject to	$\boldsymbol{x}\in\boldsymbol{X}$

$\alpha=0$ - utilitarian solution (system optimum, max-flow)

$\alpha=1$ - proportional fairness

$\alpha\rightarrow\infty$ - max-min fairness



[H.Luss, Equitable Resource Allocation: Models, Algorithms and Applications, John Wiley & Sons, 2012]

1. Theoretical background

$$U_j(x_j,\alpha)=\begin{cases}\frac{x_j^{1-\alpha}}{1-\alpha} \; for\, \alpha\geq 0, \alpha\neq 0 \\ \log(x_j)\, \alpha=1\end{cases}$$

Utilitarian solution (system optimum, max-flow):

- $\alpha=0$

The utilitarian solution maximises aggregate utility and hence it is important benchmark .

The utilitarian solution can leave some users with zero allocation and that can lead to large inequality. This can be considered unfair from the user point of view.

Property of the utilitarian solution: To increase an allocation by a ε, we have to decrease a set of other allocations, such that the sum of of decreases is larger or equal to ε.

1. Theoretical background

$$U_j(x_j,\alpha)=\begin{cases}\frac{x_j^{1-\alpha}}{1-\alpha} for\, \alpha\geq 0, \alpha\neq 0\\ \log(x_j)\, \alpha=1\end{cases}$$

Proportional fairness:

- $\alpha=1$

The proportionally fair allocation x_j^{PF} satisfies: $\sum_{j=1}^{n}\frac{x_j-x_j^{PF}}{x_j^{PF}}\leq 0$

To increase the allocation by a percentage ε, we have to decrease a set of other allocations, such that the sum of of percentage decreases is larger or equal to ε.

In communication networks, proportional fairness has emerged as a compromise between efficiency and fairness with an attractive interpretation in terms of shadow prices and market clearing mechanism.

[Frank P Kelly, Aman K Maulloo, and David KH Tan. “Rate control for communication networks: shadow prices, proportional fairness and stability”. In: Journal of the Operational Research society 49 (3) (1998), pp. 237–252.]

1. Theoretical background

$$U_j(x_j,\alpha)=\begin{cases}\dfrac{x_j^{1-\alpha}}{1-\alpha} \; for\, \alpha\geq 0, \alpha\neq 0 \\ \log(x_j) \alpha=1\end{cases}$$

Max-min fairness:

- $\alpha \rightarrow \infty$

- Intuitively, max-min fairness **recursively maximises the minimum utility**.
- Allocation is max-min fair if **no utility can be improved without simultaneously decreasing another utility that is already smaller** than or equal to the former (wealthy can only get wealthier by making poor even poorer).
- The computations involve a sequential optimisation procedure that identifies the corresponding utility levels at each step.

- Generalisation of the approach to discrete problems: Lexicographic minimax. From the vector $U(x)=(U_1(x),U_2(x),\ldots,U_n(x))$ we generate the vector $U^{(n)}(x)=(U_{j_1}(x),U_{j_2}(x),\ldots,U_{j_n}(x))$ by sorting performance functions in nonincreasing order. Then, we aim to find the solution corresponding to the lexicographically minimal vector.

Application I: Resilience of Natural Gas Networks During Conflicts, Crises and Disruptions

CRITICALITIES

EXPOSURE

RESILIENCE

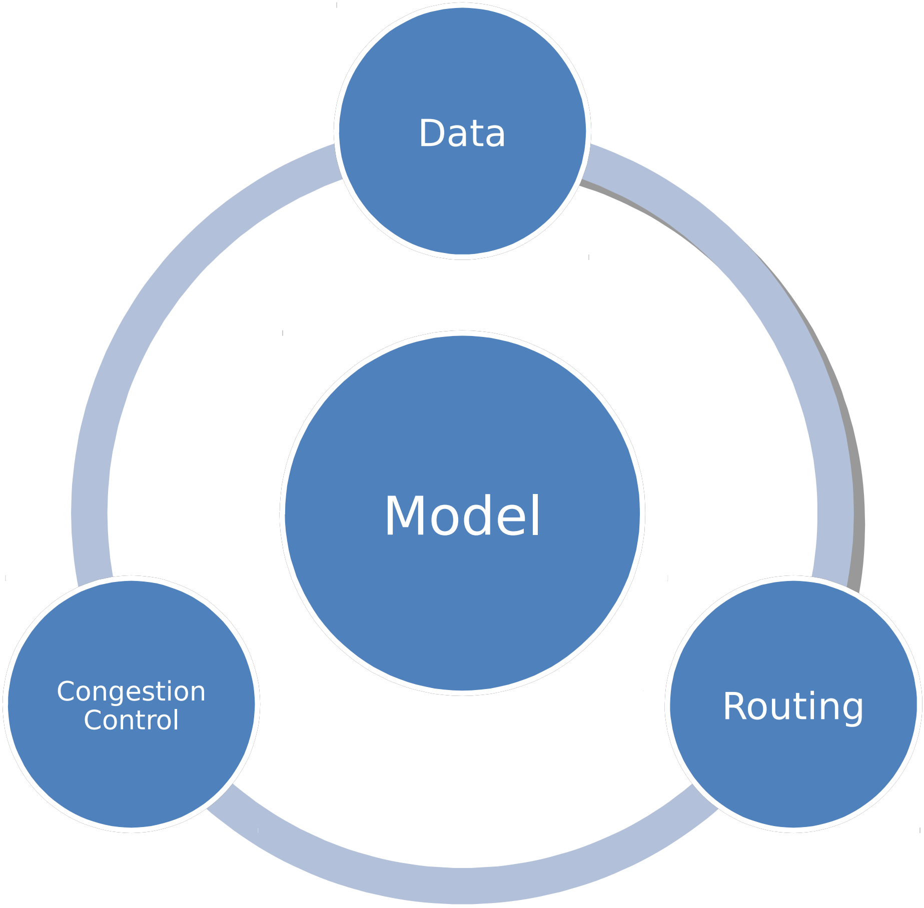
- Historical dependency of gas supply from few large sources leaves the European continent exposed to both a pipeline network that was not designed to transport large quantities of gas imported via Liquefied Natural Gas (LNG) terminals,
- and to the effects of **political and social instabilities** in countries that are heavily dependent either on the export of natural gas (eg Algeria, Libya, Qatar or Russia) or its transit (eg. Ukraine).

Hence, it is challenging to **build infrastructure that will be resilient** to a wide range of possible crisis scenarios.

(joint work with Rui Carvalho, University of Durham, Flavio Bono, Marcelo Masera, Joint Research Center of EC, D. K. Arrowsmith, QMUL, Dirk Helbing, ETH Zurich

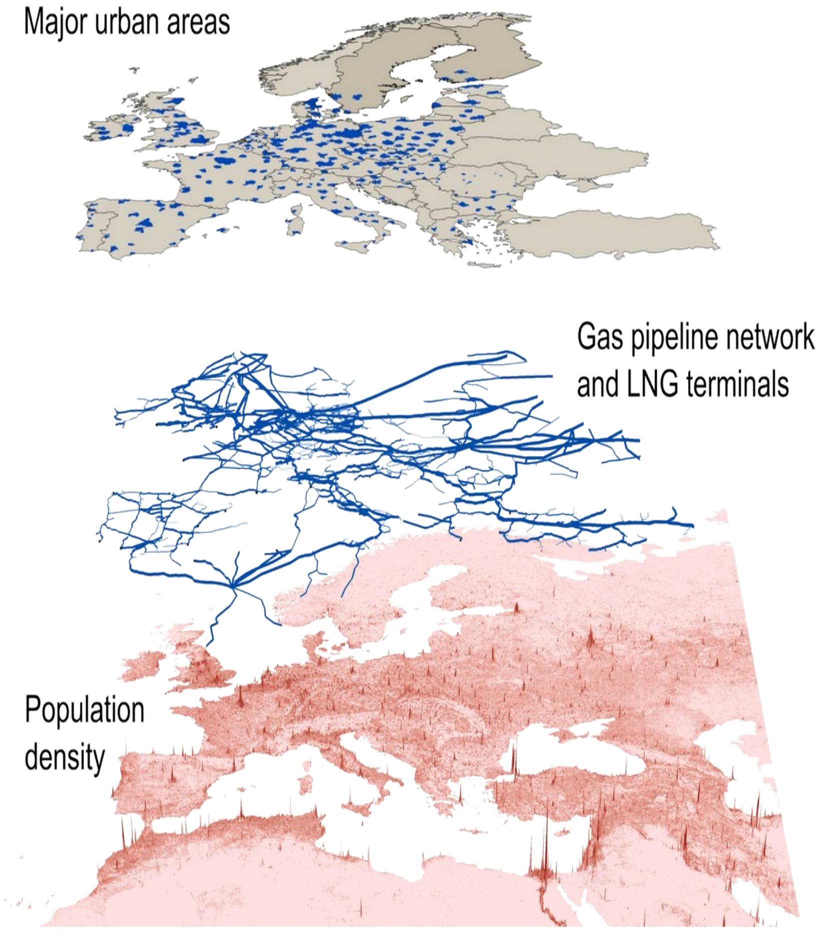
R. Carvalho, L. Buzna, F. Bono, M. Masera, D. K. Arrowsmith, D.Helbing, Resielience of Natural Gas Networks during Conflicts, Crises, and Disruptions, PLoS ONE 9(3): e90265, 2014

Methodology



Data

GIS data



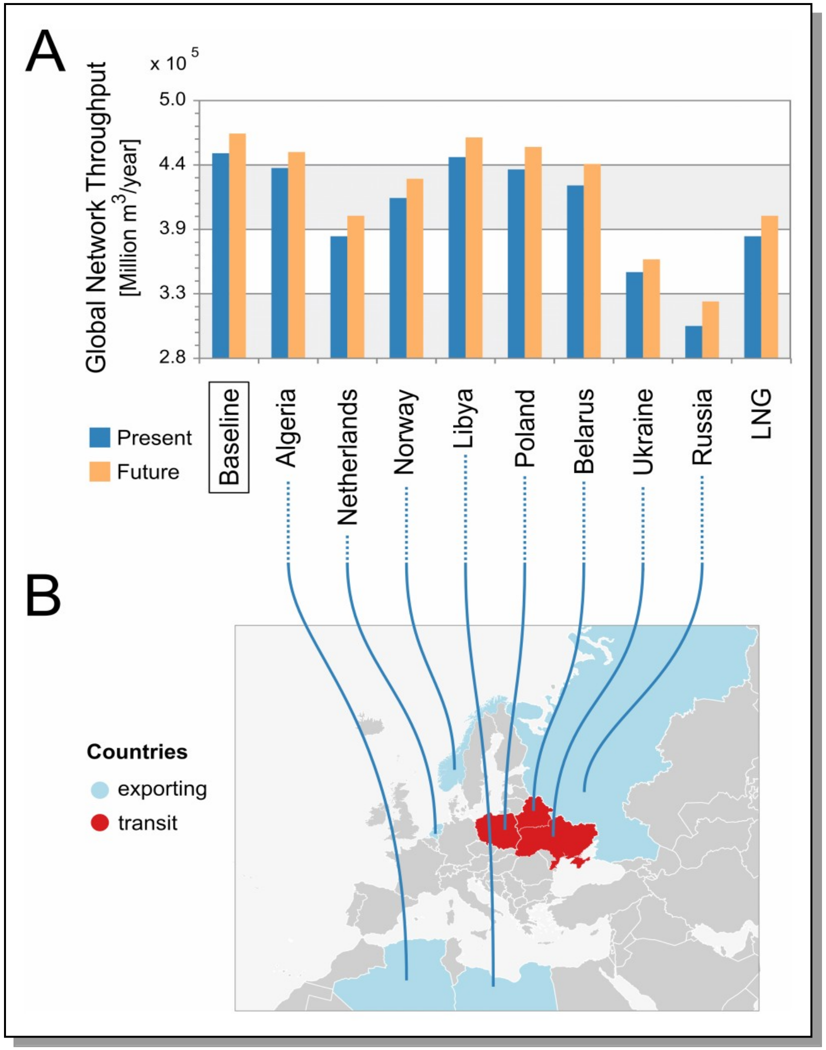
Gas trade network

To find the proportional fair allocation, we need to maximize $U(f)$, constrained to the vector of path flows being feasible:

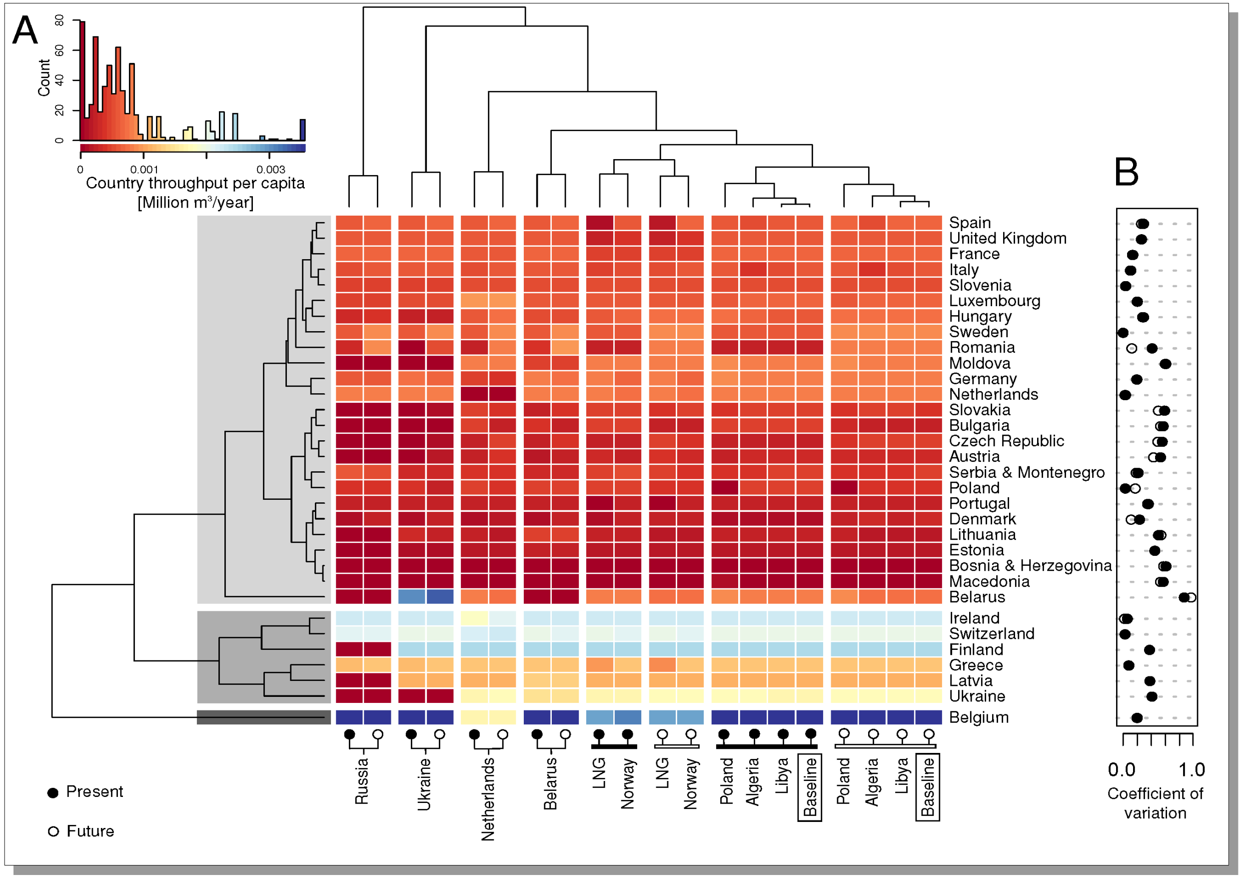
$$\begin{aligned} \underset{f}{\text{maximize}} \quad & U(f) = \sum_{j=1}^{\rho} \log(f_j) \\ \text{subject to} \quad & Bf \leq c \\ & f_j \geq 0, \end{aligned}$$

The aggregate utility $U(f)$ is concave and the inequality constraints are convex. Hence the optimization problem is convex. Thus, any locally optimal point is also a global optimum.

Results: Global network throughput by scenario

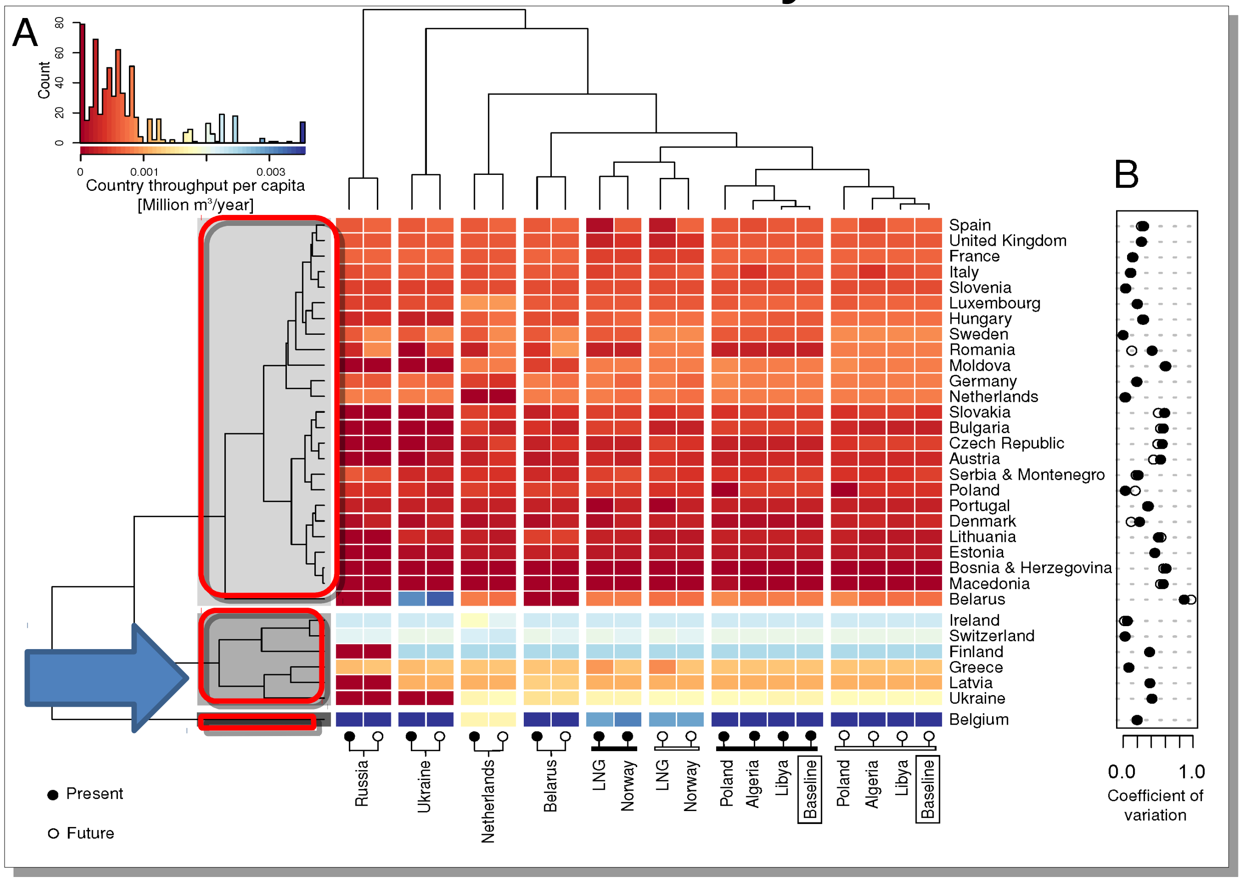


Results: Resilience at country and network levels



- A **country is resilient** to crises if it combines high throughput per capita across scenarios with a low coefficient of variation of throughput.
- The **network is resilient** to a scenario if the vectors of country throughput per capita for the scenario and the baseline scenario are similar.

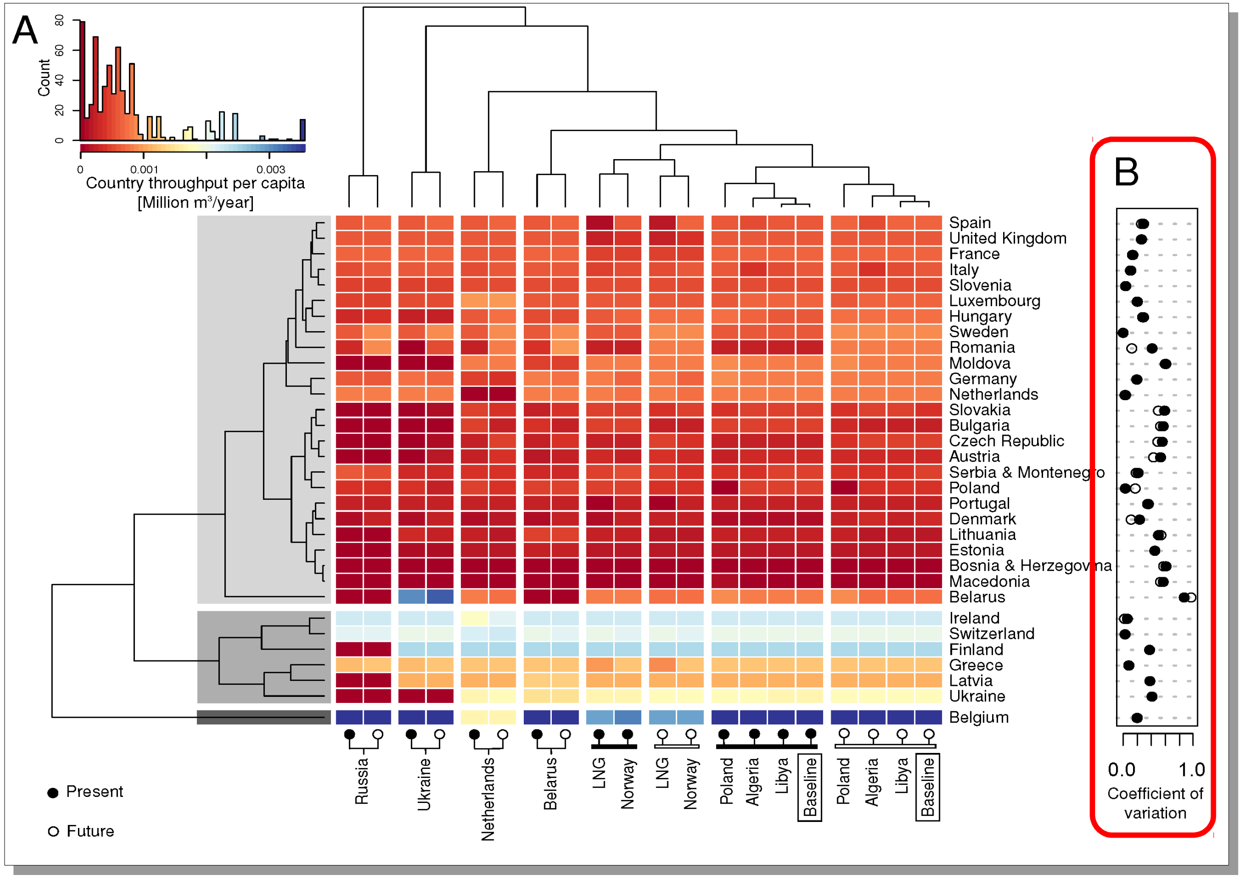
Results: Resilience at country and network levels



We analyse the country throughput per capita in the scenario space (20 scenarios) and in the country space (32 countries). The country groups highlighted in grey reflect a similar level of throughput per capita across scenarios;

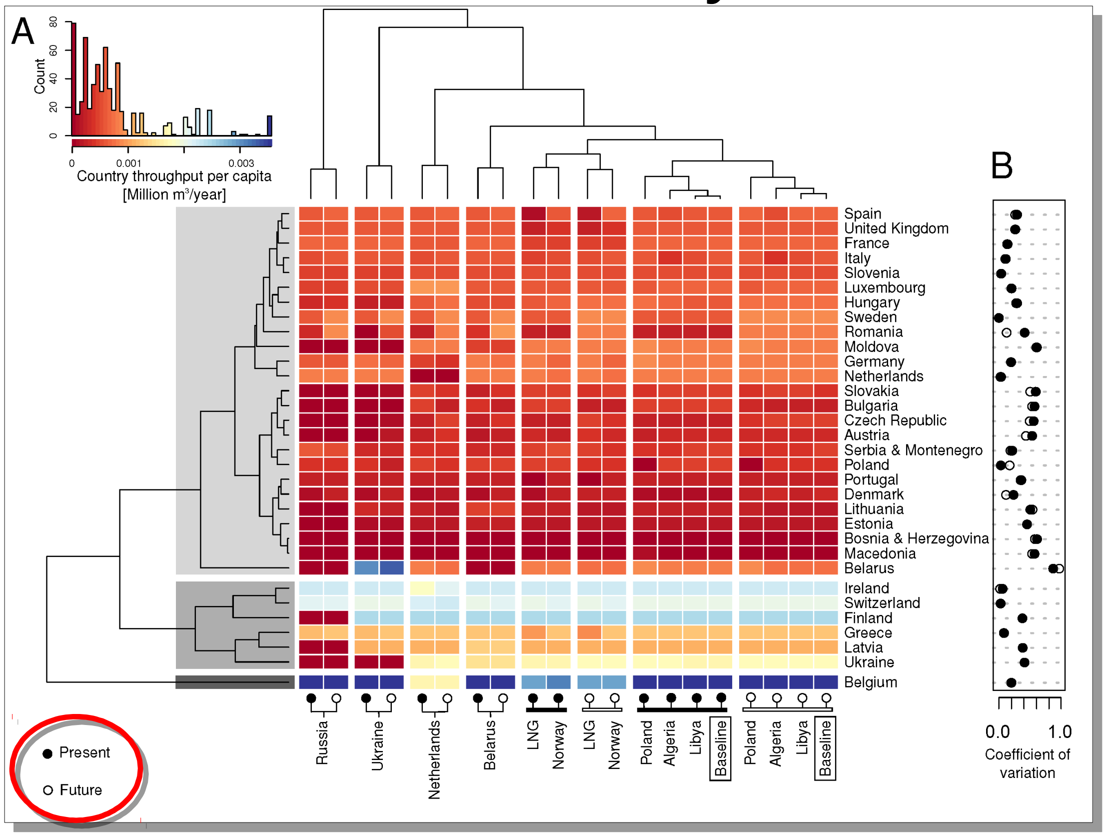
Countries belong to the high throughput per capita groups (dark grey) due to a combination of factors: diversity of supply; good access to network capacity (strategic geographical location); relatively small population.

Results: Resilience at country and network levels



- Larger values of the *coefficient of variation* indicate that country throughput varies across scenarios;
- Eastern European countries are sensitive to the scenarios where we hypothetically remove Russia or Ukraine –they are dependent on these countries;

Results: Resilience at country and network levels

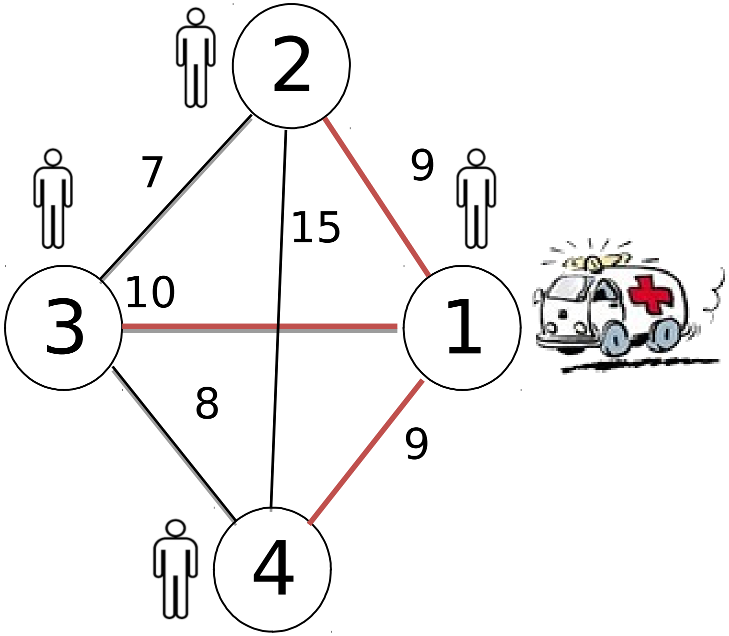


The *present* and *future* scenarios are clustered together when either Russia, Ukraine, the Netherlands, or Belarus are removed from the network;

Largest disruption

Most benign scenario

Application II: Design of Public Service Systems



Sorted distances:
1: 10,9,9,0
2: 15,9,7,0
3: 10,8,7,0
4: 15,9,8,0

Set of feasible solutions

$$\sum_{i \in I} y_i = p,$$

$$\sum_{i \in I} x_{ij} = 1 \qquad \text{for all } j \in J,$$

$$x_{ij} \leq y_i \qquad \text{for all } i \in I, j \in J,$$

$$x_{ij} \in \{0, 1\} \qquad \text{for all } i \in I, j \in J,$$

$$y_i \in \{0, 1\} \qquad \text{for all } i \in I,$$

We denote the set of all feasible location patterns, which satisfy these constraints , by the symbol Q.

Algorithm A-LEX

- Algorithm A-LEX subsequently solves optimization problems corresponding to the distance values in stages.
- We order the set of all feasible distance values $d_{i,j}$ into the descending sequence of unique distance values D_k, for $k = 1, \ldots, k_{max}$.
- At each stage $k > 1$ we consider a partitioning of the set J into the system of subsets $\{J_1, \ldots, J_{k-1}, C_k\}$, where C_k is a set of active customers.
- $J_k \subseteq C_k$ is the minimal subset of customers, whose distance from the closest facility location equals to the value D_k.

Algorithm A-LEX

For a given value of D_k, we find the minimal set J_k by solving the problem P_k:

$$\text{Minimize} \qquad g^k(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} r_{ij}^k x_{ij}$$

$$\text{Subject to} \qquad (\mathbf{x}, \mathbf{y}) \in Q.$$

Coefficients r_{ij}^k are defined for $j \in C_k$ and $i \in I$ in the following way:

$$r_{ij}^k = \begin{cases} 0, & \text{if } d_{ij} < D_k, \\ b_j, & \text{if } d_{ij} = D_k, \\ (1 + \sum_{u \in C_k} b_u), & \text{if } d_{ij} > D_k, \end{cases}$$

and for $j \in J_l$ where $l = 1, \ldots, k-1$ and $i \in I$ according to the following prescription:

$$r_{ij}^k = \begin{cases} 0, & \text{if } d_{ij} \leq D_l, \\ (1 + \sum_{u \in C_k} b_u), & \text{otherwise.} \end{cases}$$

Algorithm A-LEX

Knowing the optimal solution $(\mathbf{x^k}, \mathbf{y^k})$ of the problem P_k, the following implications denoted as cases (a), (b), and (c) can be derived:

(a) If $g^k(\mathbf{x^k}) = 0$, then each customer $j \in C_k$ can be assigned to a facility whose distance from j is less than D_k.

(b) If $0 < g^k(\mathbf{x^k}) < 1 + \sum_{u \in C_k} b_u$, then each customer $j \in C_k$ can be assigned to a facility, whose distance from j is less or equal to D_k. The minimal subset of customers $J_k \subseteq C_k$, whose distance from the closest facility locations equals to the value D_k can be defined as $\{j \in C_k | \sum_{i \in I} r_{ij}^k x_{ij}^k = b_j\}$.

(c) If $g^k(\mathbf{x^k}) > \sum_{u \in C_k} b_u$, then this case indicates non-existence of a solution $(\mathbf{x}, \mathbf{y})$ to the problem P_k, for which $\sum_{i \in I} d_{ij} x_{ij} \leq D_l$ for $j \in J_l$, where $l = 1, \ldots, k$.

Algorithm A-LEX

Algorithm A-LEX

Step 0: Initialize $k = 1$ and $C_1 = J$.

Step 1: Solve the problem P_k and denote the optimal solution by $(\mathbf{x}^\mathbf{k}, \mathbf{y}^\mathbf{k})$.

Step 2: If $g^k(\mathbf{x}^\mathbf{k}) = 0$, set $C_{k+1} = C_k$ and go to Step 4, otherwise if $(0 < g^k(\mathbf{x}^\mathbf{k}) < 1 + \sum_{u \in C_k} b_u)$ go to Step 3.

Step 3: Set $J_k = \{j \in C_k | \sum_{i \in I} r_{ij}^k x_{ij}^k = b_j\}$; $C_{k+1} = C_k - J_k$.

Step 4: If $C_{k+1} = \emptyset$, then terminate and return $(\mathbf{x}^\mathbf{k}, \mathbf{y}^\mathbf{k})$ as the solution, otherwise set $k = k + 1$ and continue with Step 1.

A necessary condition for obtaining an approximate solution is that at the stage k there exist at least two optimal solutions of the problem P_k.

Numerical experiments

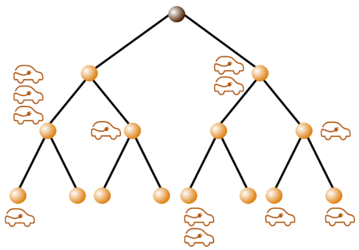
Instance	$\|I\|$	p	k_{max}	O-LEX Time [s]	O-LEX k_s	A-LEX^X Time [s]	A-LEX^X k_s	Δ
$SJC2$	200	10	426	131.4	238	50.9	237	0
$SJC2$	200	20	306	64.4	132	37.4	128	0
$SJC2$	200	30	218	32.2	79	17.4	71	0
$SJC2$	200	40	169	20.3	39	9.7	40	0
$SJC3$	300	15	445	461.6	207	357.7	189	0
$SJC3$	300	30	267	145.1	70	68.8	58	0
$SJC3$	300	45	226	71.1	46	37.9	41	0
$SJC3$	300	60	215	53.3	50	29.8	42	0
$SJC4$	402	20	461	1371.2	161	1205.8	140	0
$SJC4$	402	40	342	1207.5	74	1052.5	58	0
$SJC4$	402	60	229	158.7	33	87.2	29	0
$SJC4$	402	80	193	144.9	25	56.2	24	0
$Spain\_737\_1$	737	37	467	116838.0	92	81185.1	65	0
$Spain\_737\_1$	737	50	348	196000.0	53	27296.2	49	0
$Spain\_737\_1$	737	185	108	12367.4	5	279.5	5	0
$Spain\_737\_1$	737	259	59	430.4	2	32.2	2	0
$Spain\_737\_2$	737	37	467	35590.7	88	29185.6	65	0
$Spain\_737\_2$	737	50	348	64005.7	59	27806.1	38	0
$Spain\_737\_2$	737	185	108	3182.3	5	232.4	5	0
$Spain\_737\_2$	737	259	59	72.5	1	43.2	1	2

Instance	$\|I\|$	p	k_{max}	A-LEX^Z Time [s]	A-LEX^Z k_s
$p3038$	3038	2500	33	2520.8	1
$p3038$	3038	2000	35	4204.3	2
$p3038$	3038	1500	38	8915.9	2
$p3038$	3038	900	73	190092.9	7
$p3038$	3038	700	110	17902.9	8
$p3038$	3038	100	*	*	*
$p3038$	3038	50	*	*	*
$p3038$	3038	10	1188	201165.3	36
SR	2928	2500	3	786.8	0
SR	2928	2000	4	1021.6	0
SR	2928	1500	5	1083.4	0
SR	2928	1000	8	1622.9	0
SR	2928	900	9	1988.5	0
SR	2928	700	11	2954.2	0
SR	2928	100	48	9624.7	0
SR	2928	50	65	10509.8	0
SR	2928	10	92	11888.4	0
US	2398	2000	10	1006.6	0
US	2398	1500	15	1203.7	0
US	2398	1000	22	1702.7	0
US	2398	900	25	1694.7	0
US	2398	700	33	11022.5	0
US	2398	100	*	*	*
US	2398	50	*	*	*
US	2398	10	219	9038.2	0

- The algorithm found the optimal solution for all instances, where we could compare the results with the exact algorithm and it allows for **solving larger problems**
- The algorithm A-LEX computed all small instances in the time which accounts for **42.5%** and all medium instances for **47.6%** of the time needed by the algorithm O-LEX
- Proposed approximation method is **applicable to other types of similar problems** with lexicographical minimax objective (e.g. maximum generalised assignment problem)

W. Ogryczak, On the lexicographic minmax approach to location problems, European Journal of Operational Research 100 (1997) 566–585.

Application III: Coordination of EVs Charging in the Distribution Networks



Research question: Which congestion control algorithm(s) should be implemented in electrical distribution networks?

Simulation model:

- arrival process (Poisson process),
- congestion management protocols (proportional fairness, max-flow).

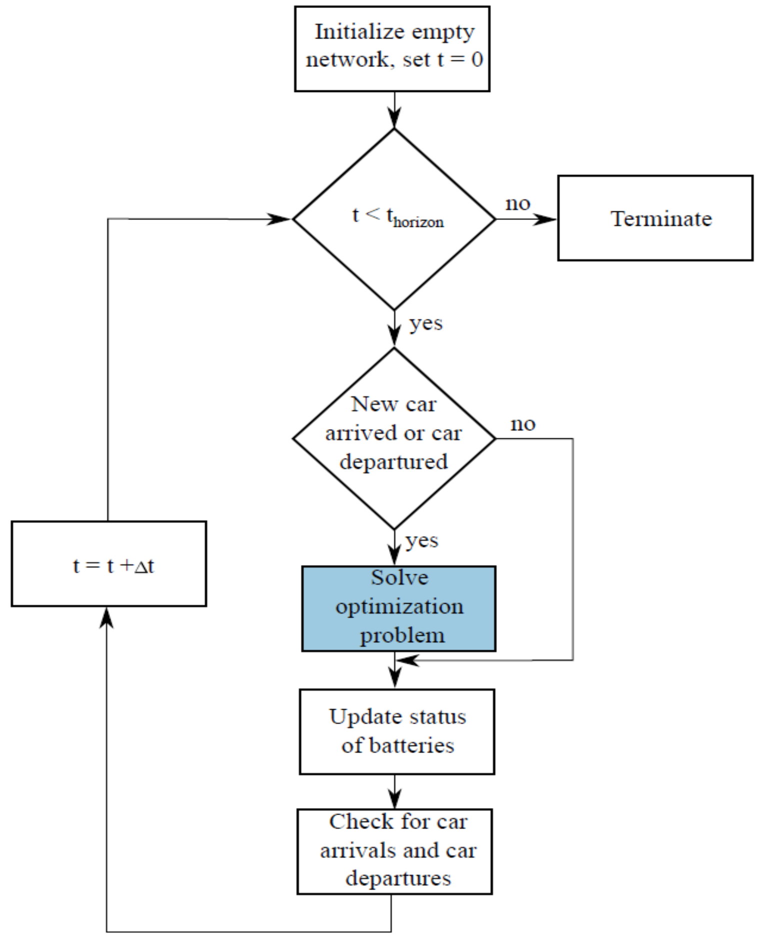
Results:

- If the inter-arrival time becomes too short, the charging of vehicles takes too long, and at some point more cars arrive for charging than leave fully charged. Hence at the critical value of the arrival rate the system undergoes a continuous phase transition from free-flow phase to congested phase.
- We study numerically the critical value of the arrival rate for realistic networks.
- We validate our findings by analysing critical arrival rate on small network for the simplified setup.

(joint work with Rui Carvalho, University of Durham, Richard Gibbens and Frank Kelly, University of Cambridge)

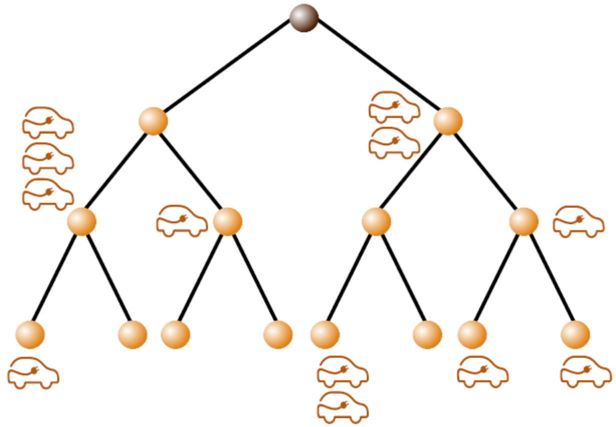
R. Carvalho, L. Buzna, R. Gibbens, F. Kelly, Critical behaviour in charging of electric vehicles, New J. Phys. (17): 095001, 2015

Simulation model: Discrete simulator



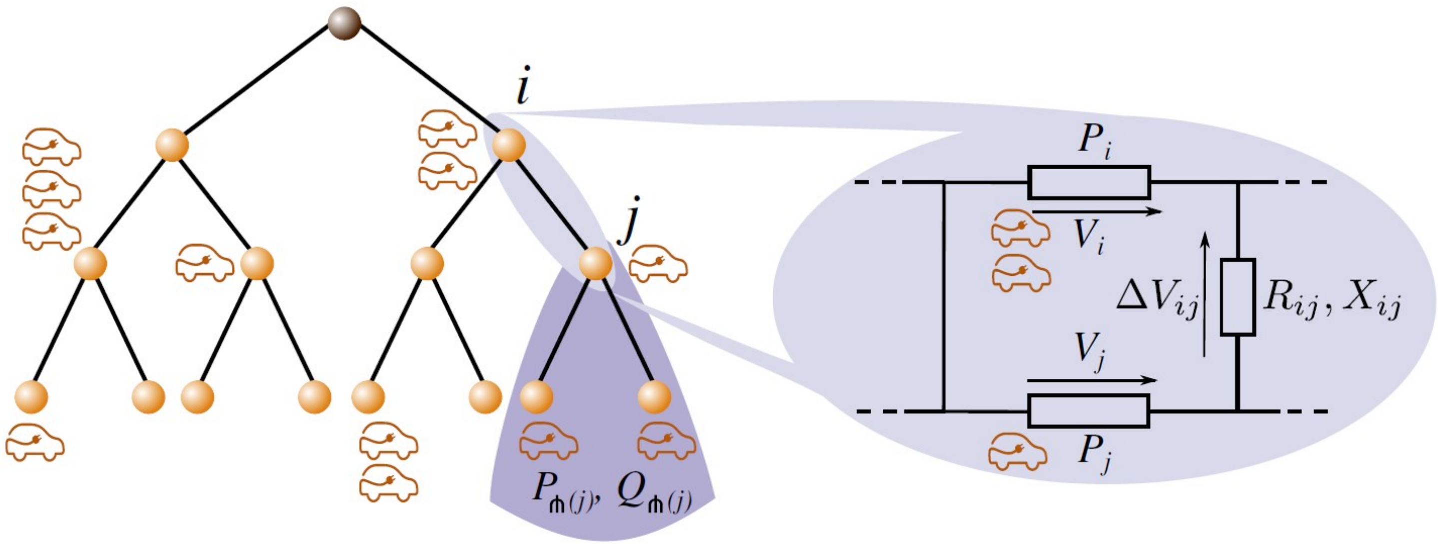
Python
cvxopt solver

Optimization model: Modelling assumptions



- Electric vehicles arrive (Poisson process) with empty batteries,choose with uniform probability to connect to a node and leave when fully charged.
- Tree-like network topology (distribution networks) → No need to apply Kirhhoff's voltage law on network loops.
- One feeder node.
- Batteries of vehicles are modelled as elastic loads (i.e. able to absorb any value of power they are allocated by the network) and the state of the battery is time integral of allocated power.

Optimization model: Voltage drop



$$\Delta V_{ij} = |V_i| - |V_j| \simeq V_i - V_j =$$

$$= \Re\left(I_{ij}\, Z_{ij}\right) = \Re\left(\frac{I_{ij}\, Z_{ij}\, V_j^*}{V_j^*}\right) = \Re\left(\frac{S^*_{\pitchfork(j)}\, Z_{ij}}{V_j^*}\right) =$$

$$= \Re\left(\frac{(P_{\pitchfork(j)} - iQ_{\pitchfork(j)})(R_{ij} + iX_{ij})}{V_j^*}\right) \simeq \frac{P_{\pitchfork(j)} R_{ij} + Q_{\pitchfork(j)} X_{ij}}{V_j},$$

From here we get:

$$V_i(t)V_j(t) - V_j^2(t) - P_{\pitchfork(j)}(t)R_{ij} - Q_{\pitchfork(j)}(t)X_{ij} = 0$$

Optimization model

$$\underset{V(t)}{\text{maximise}} \quad U(t) = \sum_{l=1}^{N(t)} U_l(P_l(t))$$

$$\text{subject to} \quad (1-\alpha)V_{nominal} \le V_i(t) \le (1+\alpha)V_{nominal}, \qquad i \in \mathcal{V}$$

$$V_i(t)V_j(t) - V_j(t)V_j(t) - P_{\pitchfork(j)}(t)R_{ij} - Q_{\pitchfork(j)}(t)X_{ij} = 0, \qquad e_{ij} \in \mathcal{E}$$

$$P_l(t) \ge 0 \qquad l = 1, \ldots, N(t)$$

Max-flow: $U_l(P_l(t)) = P_l(t)$

Proportional fairness: $U_l(P_l(t)) = log(P_l(t))$

The first set of constraints sets the limits on the nodal voltage.

The second set of constraints is the physical law coupling the voltage to power for a subtree.

It is quadratic, hence it is not convex! So we make a substitution of variables (SD relaxation).

Optimization model: Convex relaxation

$$W(e_{ij},t) = \begin{pmatrix} V_i(t) \\ V_j(t) \end{pmatrix} \begin{pmatrix} V_i(t) & V_j(t) \end{pmatrix} = \begin{pmatrix} V_i^2(t) & V_i(t)V_j(t) \\ V_i(t)V_j(t) & V_j^2(t) \end{pmatrix} = \begin{pmatrix} W_{ii}(t) & W_{ij}(t) \\ W_{ji}(t) & W_{jj}(t) \end{pmatrix}$$

We replace variables *V* with *W* variables and to make the substitution equivalent we add for each matrix $W(e_{ij},t)$ the constraints that ensure that it is rank one and positive semidefinite.

Rank one constraint is not convex! However, (in this case) removing rank one constraint does not change the Pareto frontier or the optimum.

[L. Gan, N. Li, U. Topcu, S. Low, Exact Convex Relaxation of Optimal Power Flow in Radial Networks, IEEE Trans. Autom. Control, **60**, 72-87, 2012]

Optimization model

$$\underset{W(t)}{\text{maximise}} \quad U(t) = \sum_{l=1}^{N(t)} U_l(P_l(t))$$

$$\text{subject to} \quad ((1-\alpha)V_{\text{nominal}})^2 \le W_{ii}(t) \le ((1+\alpha)V_{\text{nominal}})^2, \qquad i \in \mathcal{V}$$

$$W_{ij}(t) - W_{jj}(t) - P_{\pitchfork(j)}(t)R_{ij} - Q_{\pitchfork(j)}(t)X_{ij} = 0, \qquad e_{ij} \in \mathcal{E}$$

$$W(e_{ij}, t) \ge 0, \qquad e_{ij} \in \mathcal{E}$$

$$P_l(t) \ge 0 \qquad l = 1, \ldots, N(t)$$

Max-flow: $U_l(P_l(t)) = P_l(t)$

Proportional fairness: $U_l(P_l(t)) = log(P_l(t))$

Numerical experiments

For Poisson arrival rate when $\lambda < \lambda_c$, all vehicles that arrive with empty batteries within a large enough time window, leave fully charged within that period (**free flow phase**);

For $\lambda > \lambda_c$, some vehicles have to wait for increasingly long times to fully charge (**congested phase**);

We characterise this behaviour by order parameter η that represents the ratio at the steady state between the uncharged vehicles and the number of vehicles that arrive to be charged:

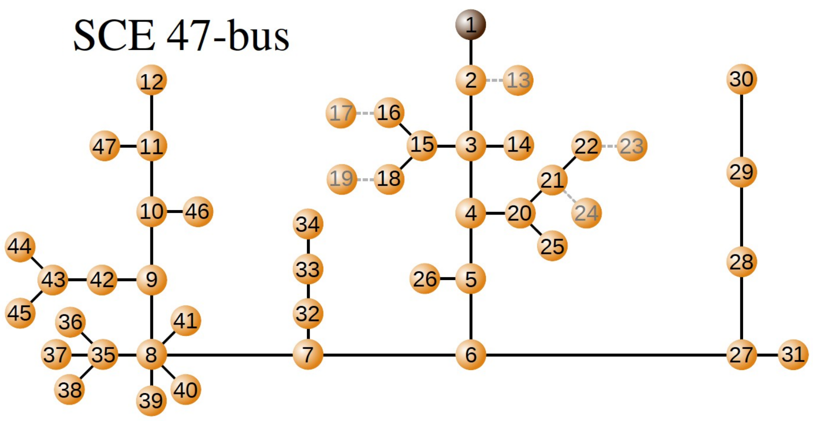
$$\eta(\lambda) = \lim_{t\to\infty} \frac{1}{\lambda} \frac{\langle \Delta N(t) \rangle}{\Delta t}$$

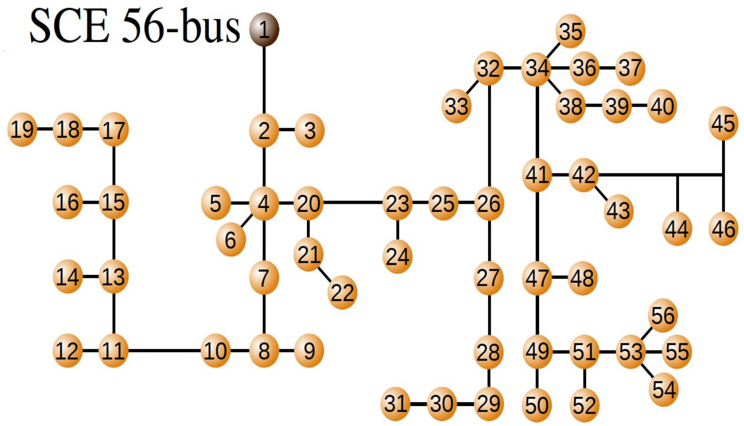
where $\Delta N(t) = N(t + \Delta t) - N(t)$ and $\langle \ldots \rangle$ indicates an average over time Δt.

$\eta(\lambda) = 0$ in the free-flow phase, up to some critical arrival rate λ_c whereas congested phase is identified by $\eta(\lambda) > 0$ and $\lambda > \lambda_c$.

[A. Arenas, A. Diaz-Guilera, R. Guimera, Communication in networks with hierarchical branching, Phys. Rev. Lett. **86**, 3196-3199, 2001]

SCE networks



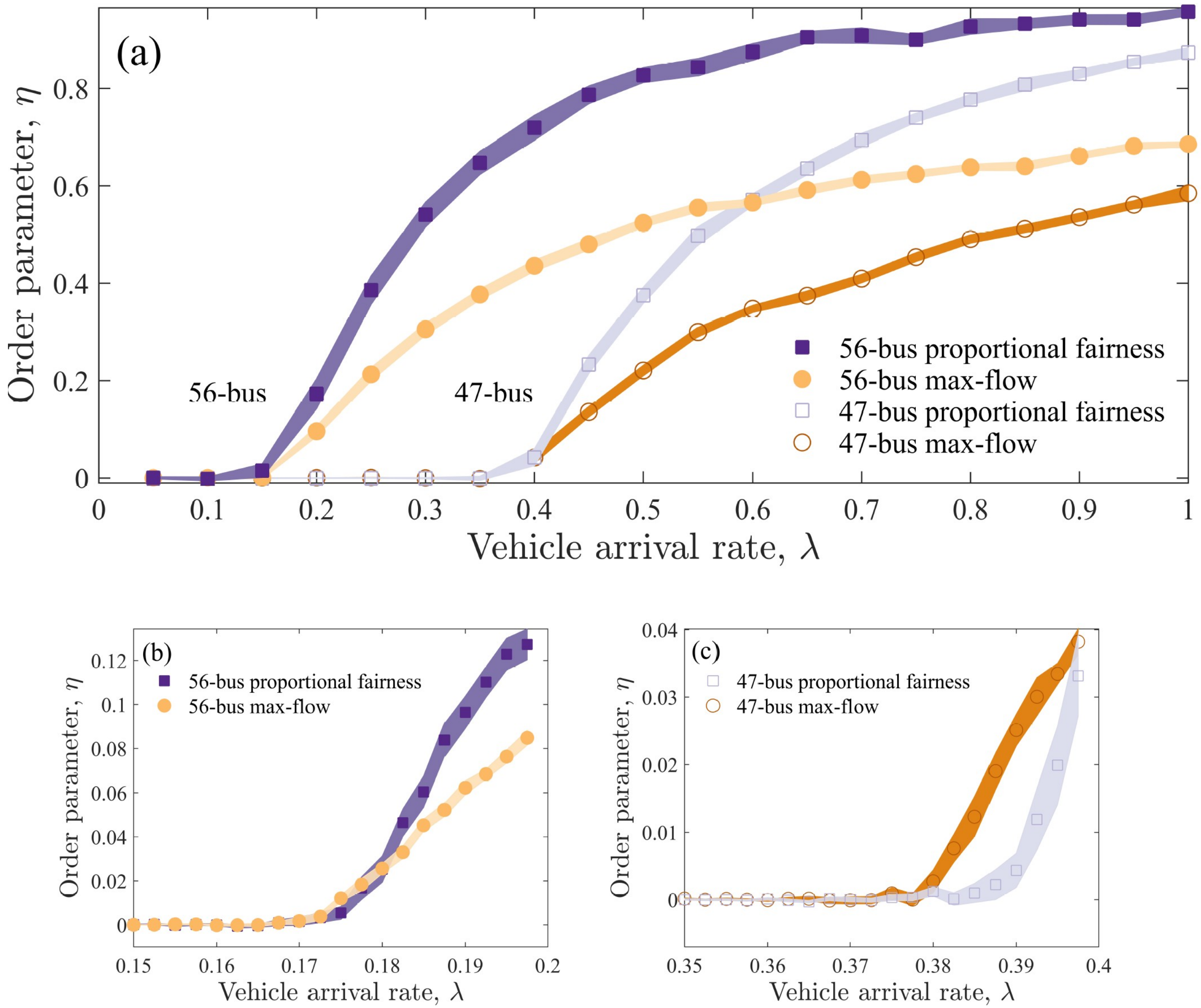


Node indexes identify the edges and resistance and reactance is taken from reference*. Node 1 is the root node in both networks. Nodes 13, 17, 19, 23, 24 (in lighter colour) are photovoltaic generators and we removed them from the network.

We set $V_{nominal} = B = 1.0$ and $\alpha = 0.1$.

*[L. Gan, N. Li, U. Topcu, S. Low, Exact Convex Relaxation of Optimal Power Flow in Radial Networks, IEEE Trans. Autom. Control, **60**, 72-87, 2012**]**

Onset of congestion



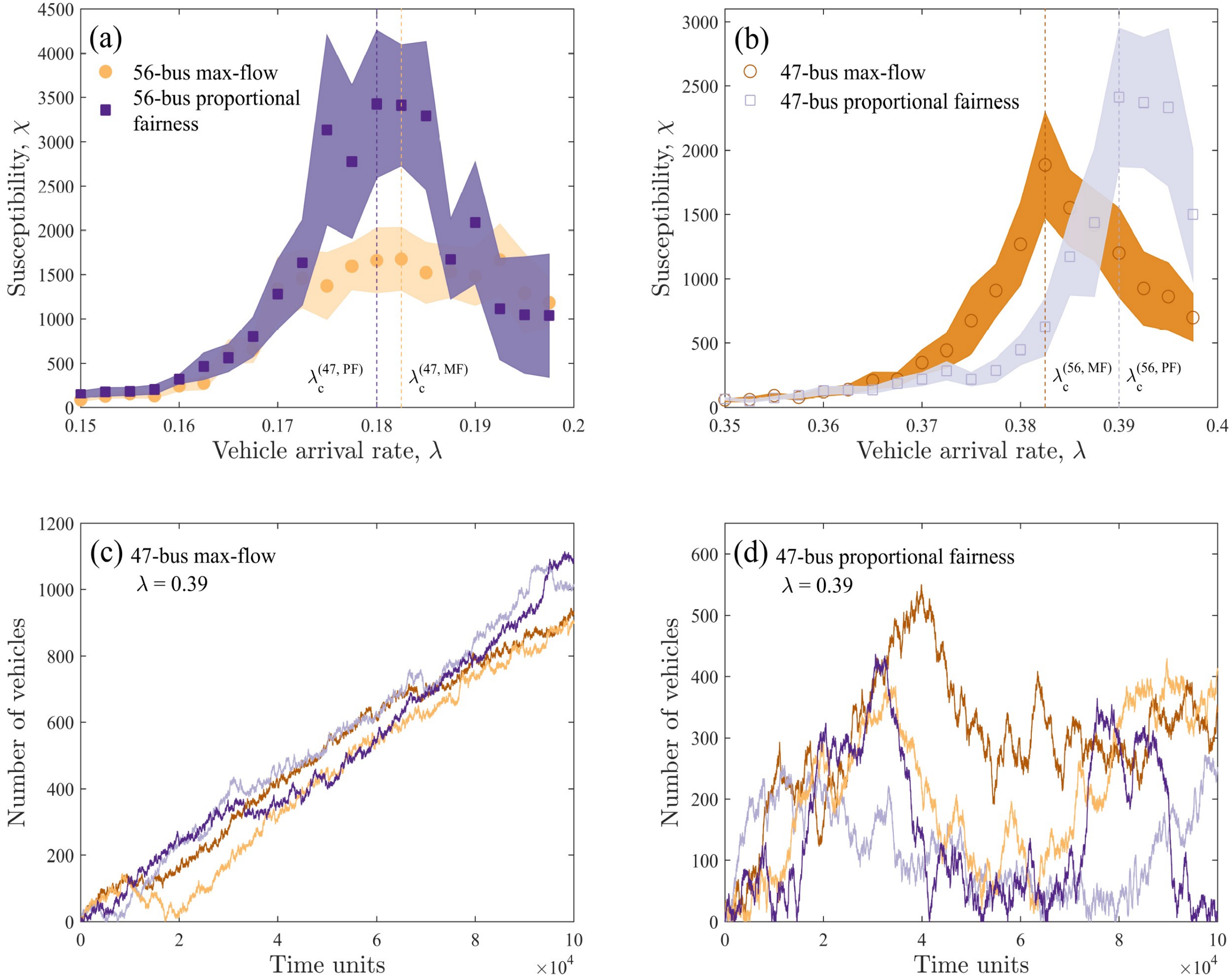
Onset of congestion

The number N(t) of charging vehicles at time t may fluctuate widely close to the critical point, and thus it is not easy to determine λ_c. To alleviate this limitation we adopt the susceptability-like measure*: $\chi(\lambda) = \lim_{\Delta t \to \infty} \Delta t \, \sigma_\eta(\Delta t)$

where Δt is the length of a time window and $\sigma_\eta(\Delta t)$ is the standard deviation of the order parameter η . To compute $\chi(\lambda)$, we first consider a long time series and split it into windows with length Δt. We next determine the value of order parameter in each window, and finally calculate the standard deviation of these values.

*[A. Arenas, A. Diaz-Guilera, R. Guimera, Communication in networks with hierarchical branching, Phys. Rev. Lett. **86**, 3196-3199, 2001]

Onset of congestion



Inequality in charging time

We compute Gini coefficient of charging time.

Gini coefficient is typically used as a measure of inequality in income distribution. Gini coefficient is one half of the mean difference in units of the mean:

$$G = \frac{1}{2\mu} \mathrm{E}\left[|u - v|\right] = \frac{1}{2\mu} \int_0^\infty \int_0^\infty |u - v| \, f(u) f(v) \, du \, dv$$

where *u* and *v* are independent identically distributed random variables with probability density f and mean μ.

For a sample (x_i, i = 1, 2, … , n), the gini coefficient may be estimated by sample mean:

$$\widehat{G} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \left| x_i - x_j \right|}{2n^2\mu}$$

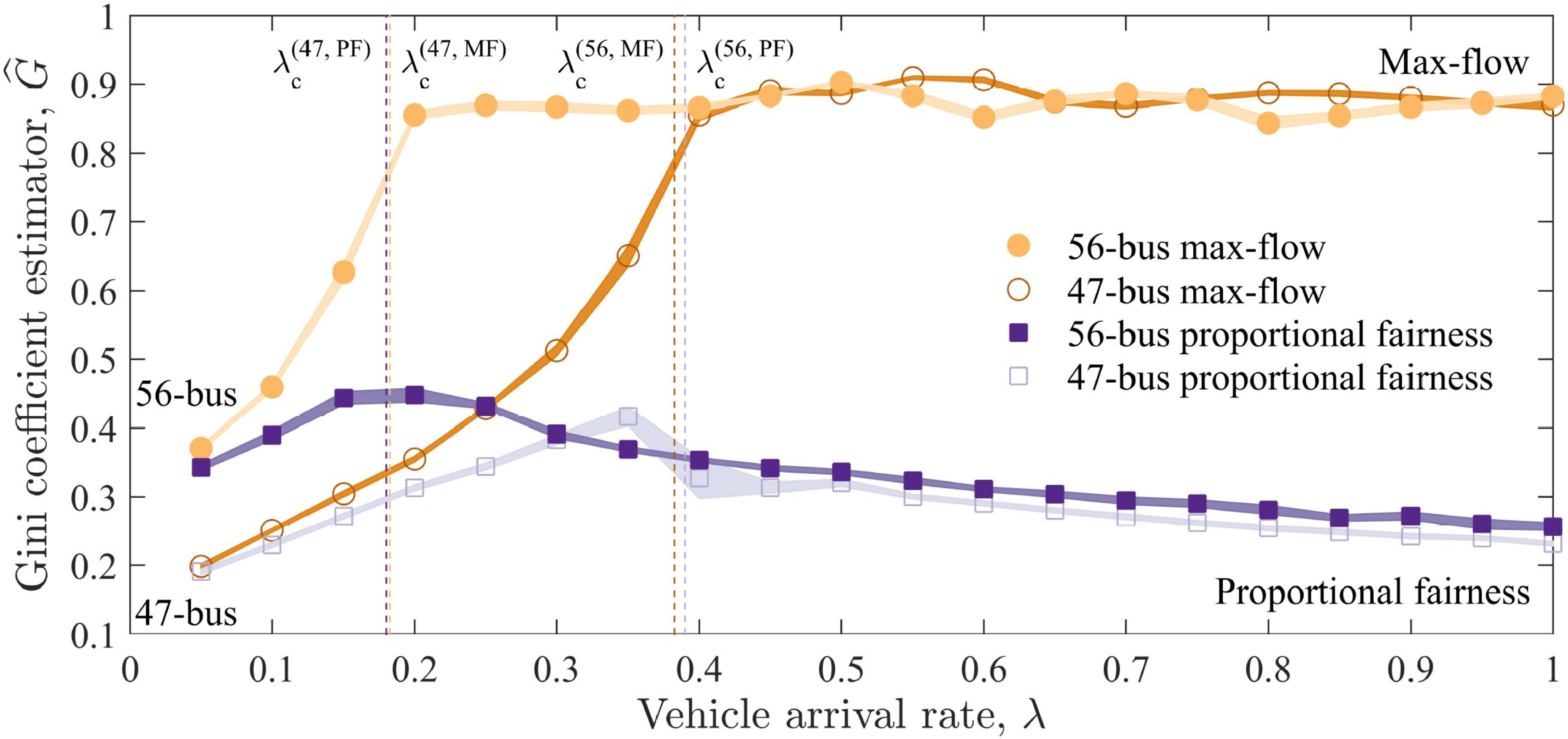
For sample with one non-zero value x we get:

$$\widehat{G} = (n - 1)/n \to 1 \text{ as } n \to \infty$$

For sample where all data points have the same value we get:

$$\widehat{G} = 0$$

Inequality in charging time



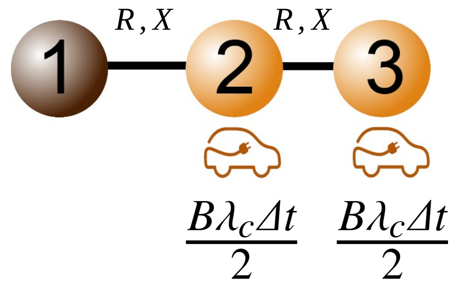
Gini coefficient values for income:

Sweden: G=0.26 U.S.: G=0.41

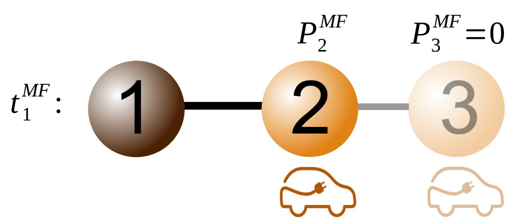
Seychelles: G=0.66

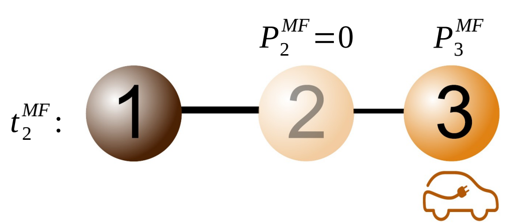
Onset of congestion in 2-edge network

We assume a time interval Δt.

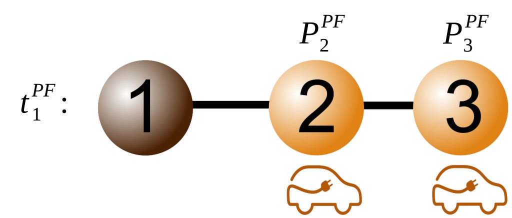


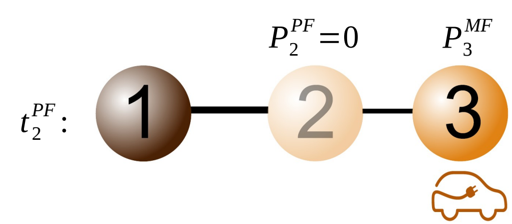
Max-flow: $\Delta t = t_1^{MF} + t_2^{MF}$





Proportional fairness: $\Delta t = t_1^{PF} + t_2^{PF}$





Onset of congestion in 2-edge network

Max-flow: $\Delta t = t_1^{MF} + t_2^{MF}$

$$P_2^{MF} t_1^{MF} = \frac{B \Lambda_c \Delta t}{2}$$

$$P_3^{MF} t_2^{MF} = \frac{B \Lambda_c \Delta t}{2}$$

Proportional fairness: $\Delta t = t_1^{PF} + t_2^{PF}$

$$P_2^{PF} t_1^{PF} = \frac{B \Lambda_c \Delta t}{2}$$

$$P_3^{MF} t_2^{PF} = \frac{B \Lambda_c \Delta t}{2} - P_3^{PF} t_1^{PF}$$

Simplification: Power allocations in PF are independent of car number, i.e. $w_1 = w_2 = 1.0$.

$$P_2^{MF} = \frac{2\alpha(1-\alpha)V_{nominal}^2}{R}$$

$$P_3^{MF} = \frac{\alpha(1-\alpha)V_{nominal}^2}{R}$$

$$P_2^{PF} = \frac{2\alpha V_{nominal}^2 (3\sqrt{\gamma} - \gamma)}{9R}$$

$$P_3^{PF} = \frac{(1-\alpha)V_{nominal}^2 (\sqrt{\gamma} + 3\alpha - 3)}{3R}$$

$$\gamma = 2\sqrt{\alpha^2 - \alpha + 1}\,|\alpha - 2| + 2\alpha^2 - 5\alpha + 5$$

$$\lambda_c^{MF} = \frac{P_2^{MF}}{\frac{B}{2}\left(\frac{P_2^{MF}}{P_3^{MF}} + 1\right)}$$

$$\lambda_c^{PF} = \frac{P_3^{MF}}{\frac{B}{2}\left(\frac{P_3^{MF}}{P_2^{PF}} - \frac{P_3^{PF}}{P_2^{PF}} + 1\right)}$$

Parameter values : R = X = B = $V_{nominal}$ = 1.0, α = 0.1.

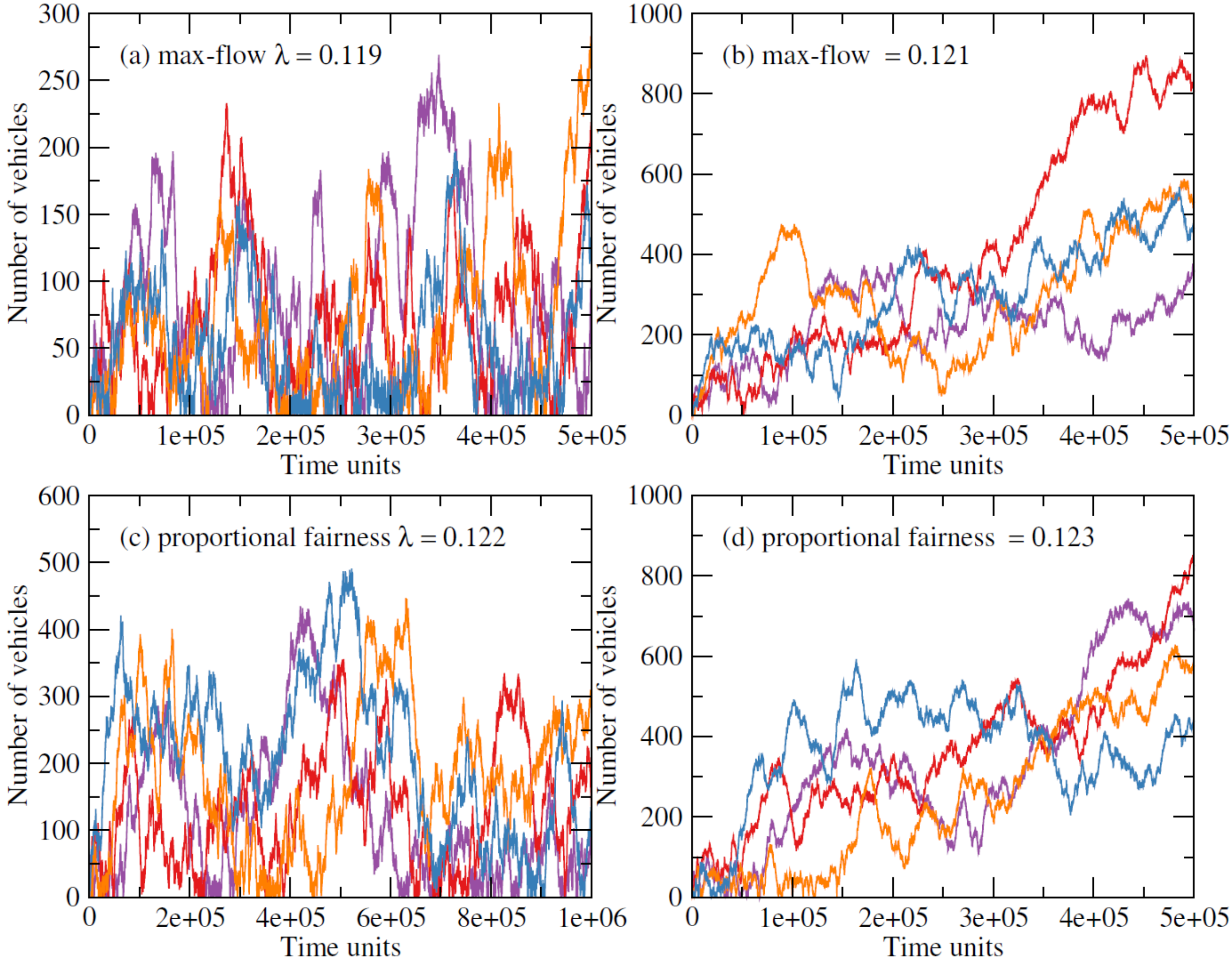
$$\lambda_c^{MF} = 0.12$$

$$\lambda_c^{PF} \approx 0.1222$$

Onset of congestion in 2-edge network

$$\lambda_c^{MF} = 0.12$$

$$\lambda_c^{PF} \approx 0.1222$$



Current and future work

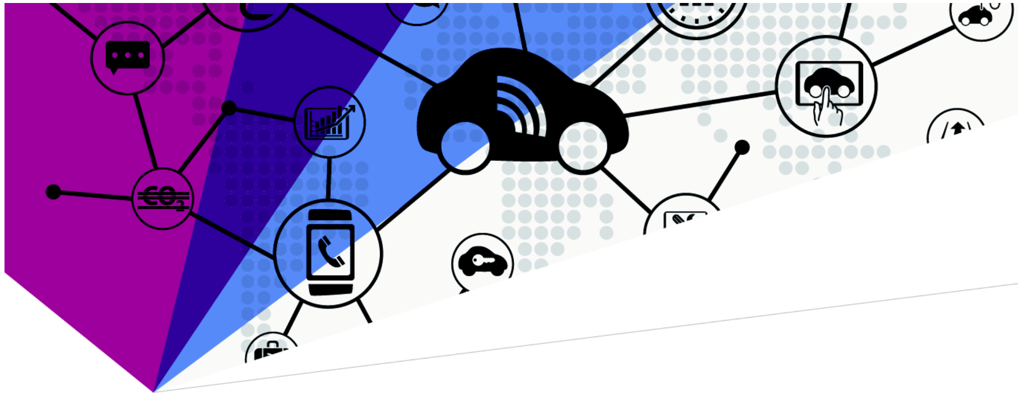
Our next goal is to study **how the future electrical distribution networks could evolve to support heterogeneous types of loads** (uncooperative and cooperative network users) **using self-organized (market) mechanism**.
The network should allow interaction between heterogeneous populations of users by providing mechanisms that could be used to provide **active network users** with necessary **information and the correct incentives** to use the network in a fair and efficient way.

What should be done ?

- reconsideration of model assumption (upper bounds for power allocations neglected reactive parts of the voltage drops, neglected thermal limits of power lines, tree-like network topology);

- enhancement of the user model (utility function, user strategy);

- enhancement of the user models using relevant field data.



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Please, feel free to ask.

Acknowledgements

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